AN942

Piecewise Linear Interpolation on PIC12/14/16 Series Microcontrollers

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INTRODUCTION

The embedded systems world is filled with sensing and measurement techniques that present non-linear output results. If the sensing technique is well characterized, a mathematical transfer function can be used to correct and linearize the sensor output. In many cases, these functions contain complex polynomial and exponential functions, placing a great burden on the program memory, RAM and execution speed of most low-cost microcontrollers.

This application note will explore a simple Piecewise Linear Interpolation technique that is often employed to linearize a sensor's output. This technique has many advantages in faster execution speed using a simple look-up table and significantly reduced program memory and RAM requirements. However, this technique can introduce a considerable error component with certain transfer functions. Therefore, the design engineer needs to understand the benefits and limitations of this technique, which will be explained in this application note.

SENSING SYSTEM INPUT

Sensors sense real-world quantities such as temperature, pressure, humidity, etc. The sensors present the phenomenon they are sensing as a voltage, current or capacitance. It is the embedded system's job to convert the sensor output into a digital value. For example, a voltage measurement is converted to a number by the microcontroller's Analog-to-Digital (A/D) Converter module, or time can be measured by the Capture/Compare/PWM (CCP) module. Once the measurement is obtained and filtered (as needed), a transfer function is applied before it is presented.

Thermistors are a classic example of a sensor with a non-linear resistance with respect to the temperature measured. The industry accepted transfer function for a thermistor follows the Steinhart-Hart equation, as shown in Equation 1. Graphically, temperature vs. resistance is plotted in Figure 1.

EQUATION 1: STEINHART-HART

$$\frac{1}{T_K} = A + B(1n\ R) + (1n\ R)3$$

Solving for temperature in degrees Celsius:

$$T_{Degrees\ C} = \frac{1}{A + B(1n\ R) + C(1n\ R)^3} - 273.15$$

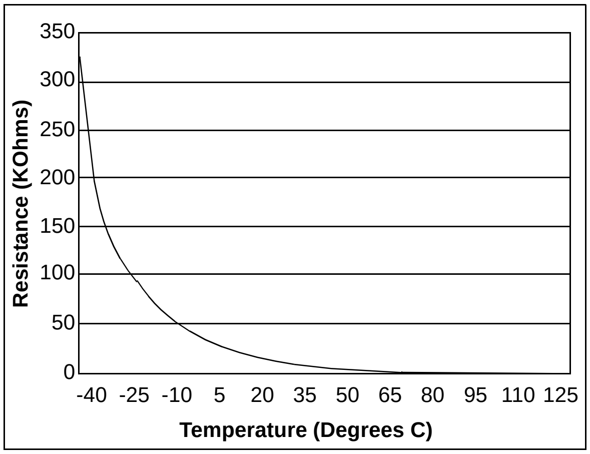
Legend:

A, B, C = Constants determined through a calibration process

R = The thermistor's resistance in ohms

T_K = The temperature in Kelvins

FIGURE 1: TEMPERATURE VS. RESISTANCE



The Steinhart-Hart equation involves 1 cube, 1 divide, 2 multiples, 2 additions, 1 subtraction and 2 logarithm math functions. This could take several thousand instruction cycles to perform each temperature measurement since most low-cost, 8-bit microcontrollers do not have divide, cube and logarithm instructions. A large amount of program memory words would also be consumed by this formula.

The transfer function for certain sensors may vary due to manufacturing tolerances in the sensor, packaging or other environmental factors. A Piecewise Linear Interpolation technique eliminates the need for complex math and enables the programming of a unique customized transfer function for each sensor to compensate for manufacturing tolerances.

WHY USE PIECEWISE LINEAR INTERPOLATION?

Piecewise Linear Interpolation (PwLI) is the technique of finding the value of a function at an unknown intermediate point given two data points. This value is calculated using a straight line between the closest two known data points.

The advantages of PwLI:

1. Fast execution speed (<137 instruction cycle).
2. Minimum program memory requirements (<248 words of program memory space including PwLI segment look-up table).
3. Optimized look-up table size (130 bytes of data for a 64 segment table).
4. No need to characterize transfer function or coefficients.
5. Transfer function can easily be tailored during manufacturing to compensate for component tolerances.

The Disadvantages of PwLI:

1. Considerable output error can be introduced if an insufficient number of segments are used.
2. Transfer function with inflection points which do not land on a segment boundary may reduce the inflection point effect.

HOW DOES PIECEWISE LINEAR INTERPOLATION WORK?

Start with a function where an output is defined by an input, as shown in Figure 2. The input can come from any source, for example, a reading from an Analog-to-Digital Controller (ADC), Timer, Capture register, or the output of a computation.

In Figure 3, two points on the function are chosen: (X1, Y1) and (X2, Y2). A line bisects the two points.

FIGURE 2: NON-LINEAR FUNCTION

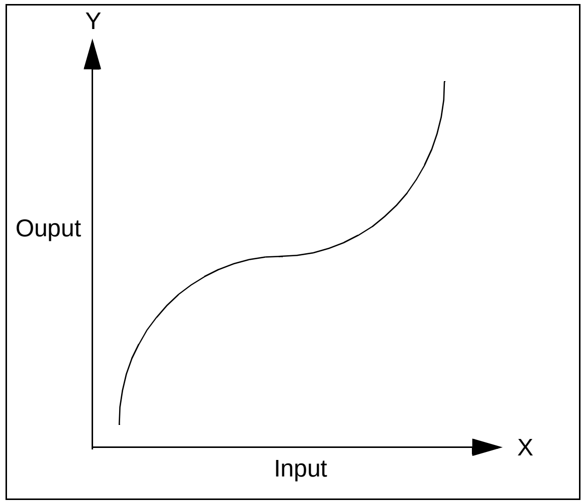
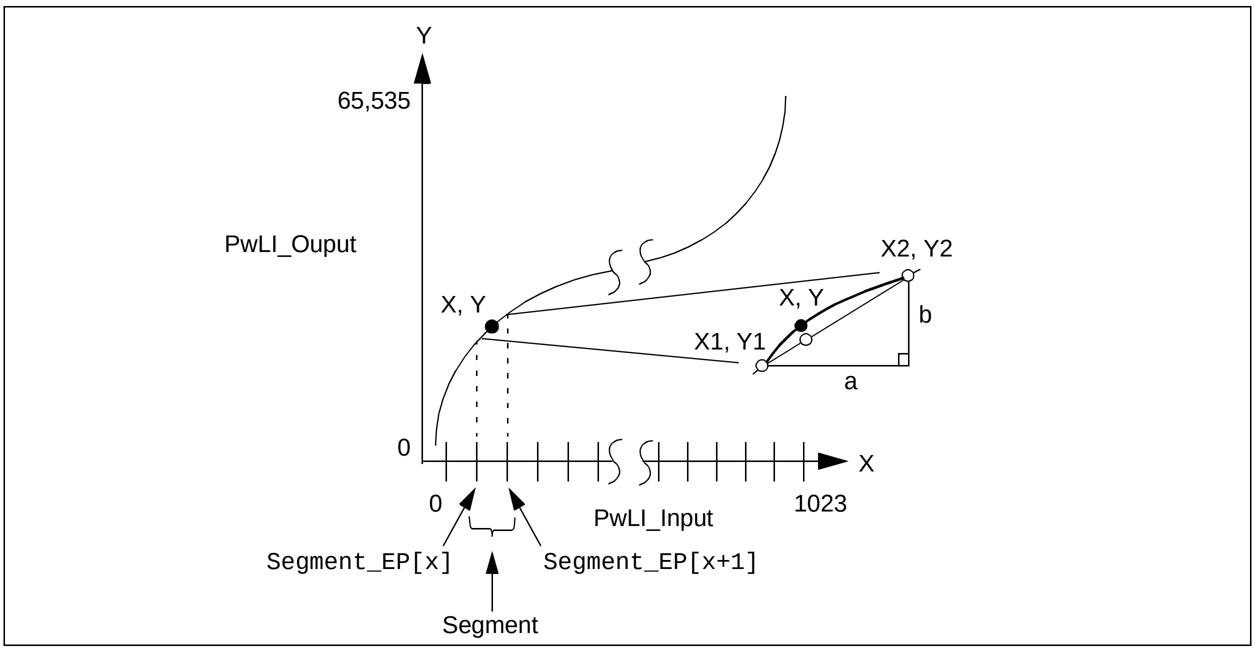
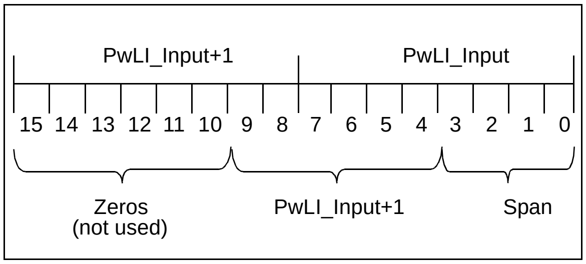


FIGURE 3: FUNCTION IS DIVIDED INTO SEGMENTS



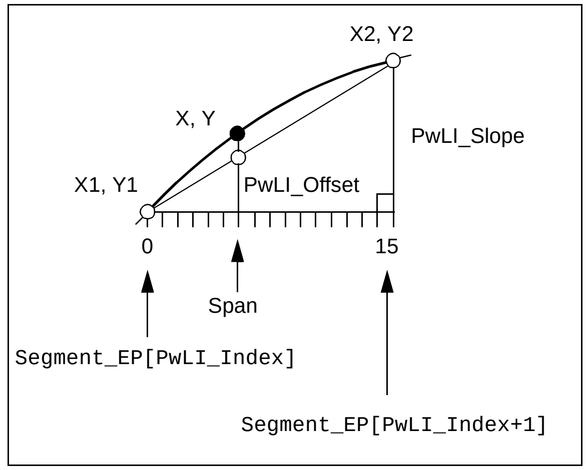
To make the best utilization of the PICmicro® devices 10-bit A/D, the input (X-axis) is divided into 64 segments and spans 10 bits (the output of a 10-bit ADC). The segments are further divided into 16 sub-segments.

FIGURE 4: THE 10-BIT ADC OUTPUT IS DIVIDED INTO SEGMENT AND SPAN VALUES



In Figure 4, the 10-bit input value contained in `PwLI_Input+1:PwLI_Input` is divided into a 6-bit segment called `PwLI_Index` and a 4-bit value called span. In Figure 5, the `PwLI_Index` value is used to reference a specific segment within a look-up table and retrieve the segment endpoint (`PwLI_Segment_EP [x]`) Y1 and Y2 (16-bit) values. An intermediate slope value is calculated by subtracting Y1 and Y2. Next, the intermediate slope is multiplied by the 4-bit `span` value and then divided by 16. The result is Y within the segment. Finally, Y is added to Y1 to complete the interpolation.

FIGURE 5: CALCULATING INTERMEDIATE POINT WITHIN THE SEGMENT



USING THE PIECEWISE LINEAR INTERPOLATION ROUTINE

The accompanying file 00942.zip contains three files: pwl_10_16.asm, pwl_10_16.inc and pwl_lookup_calculation.xls. The user must first calculate each segment endpoint and populate the Segment_EP0 through Segment_EP64 values in pwl_10_16.inc with the desired output results. The pwl_lookup_calculation.xls spreadsheet can aid in automatically generating the endpoint values and source code. Place your formula or desired output values into the PwLI_Output column. The Segment_EP0 though Segment_EP64 endpoint data are automatically calculated as shown in Table 1. Finally, cut and paste the right most column into the pwl_10_16.inc file.

TABLE 1: EXAMPLE OUTPUT FROM pwl_lookup_calculation.xls

Endpoint	PwLI Input	PwLI Output	Look-up Table data to be placed into pwl_10_16.inc file date:
0	0	0	Segment_EP0 equ 0x0000
1	16	1024	Segment_EP1 equ 0x0400
2	32	2048	Segment_EP2 equ 0x0800
3	48	3072	Segment_EP3 equ 0x0C00
62	992	63488	Segment_EP62 equ 0xF800
63	1008	64512	Segment_EP63 equ 0xFC00
64	1024	65536	Segment_EP64 equ 0xFFFF

The PwLI algorithm is utilized by first loading the `PwLI_Input` variable with the 10-bit sensor output value and calling the PwLI subroutine. The `PwLI_Output` variable will contain the 16-bit result when PwLI returns. The PwLI algorithm performs the following high-level calculations:

- Span = PwLI_Input AND 0x0F
- PwLI_Index = PwLI_Input/16
- PwLI_Slope = Segment_EP[PwLI_Index+1] – Segment_EP[PwLI_Index]
- PwLI_Offset = (PwLI_Slope * Span)/16
- PwLI_Output = Segment_EP[PwLI_Index] + PwLI_Offset

TABLE 2: PwLI VARIABLES

Variable	Size	Function
`PwLI_Input`	10-bit	PwLI input variable
`PwLI_Output`	16-bit	PwLI output variable
`PwLI_Index`	16-bit	Segment number, calculated as PwLI_Input divided by 16
`PwLI_Slope`	24-bit	Slope of segment, calculated by subtracting the segment endpoints
`PwLI_Offset`	24-bit	Result of PwLI_Slope times Span, later divided by 16

PwLI ALGORITHM PERFORMANCE AND MEMORY USAGE

A total of 248 words of program memory are used as follows:

- 130 words for storing (65) 16-bit segment endpoints
- 1 word for the endpoint look-up table "`addwf PCL`" instruction
- 2 words for PCLATH initialization
- 1 word to call PwLI code
- 114 words for PwLI algorithm

The execution time varies with the `PwLI_Input` value from 101 (minimum) to 137 (maximum) instruction cycles, including the PwLI function call and return overhead.

EXPANDING THE PwLI INPUT RESOLUTION

While this example uses a 10-bit input to generate a 16-bit output, the input resolution could be altered to accommodate 12, 14-bit or higher resolution input signal sources. The changes would be as follows:

	10-bit Input	12-bit Input	14-bit Input
Span =	PwLI_Input AND 0x0F	PwLI_Input AND 0x3F	PwLI_Input AND 0xFF
PwLI_Index =	PwLI_Input/16	PwLI_Input/64	PwLI_Input/256
PwLI_Offset =	(PwLI_Slope * Span)/16	(PwLI_Slope * Span)/64	(PwLI_Slope * Span)/256

The PwLI_Output and PwLI_Slope calculations would remain the same for all input resolution options:

- PwLI_Ouput = Segment_EP[PwLI_Index] + PwLI_Offset
- PwLI_Slope = Segment_EP[PwLI_Index+1] – Segment_EP[PwLI_Index]

SUMMARY

Piecewise Linear Interpolation is a great solution for sensor linearization due to its fast execution speed, reduced program memory requirements and ease of implementation. This technique can be expanded or simplified to include more or less segments, or provide more or less input resolution.

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